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Landau diamagnetism within Tsallis thermostatistics and quantum groups

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Abstract. In this study, using the base of coherent states, Landau diamagnetism has been generalized within Tsallis Thermostatistics. As far as we know, this is the first attempt to introduce coherent states in this formalism. The magnetization and the susceptibility of the system have been obtained and compared with the standard result to illustrate the effect of nonextensivity. Then, adding a perturbation term to the Hamiltonian of the system, nonextensive effects on diamagnetic susceptibility have been investigated. In addition to this, making use of the q_G-deformed partition function of the q_G-oscillator system, the magnetization for q_G-deformed Landau diamagnetism has been derived, with the aim of comparing the results obtained within both formalisms.

PACS. 05.20.-y Statistical mechanics – 75.20.-g diamagnetism – 03.65.-w quantum mechanics

1 Introduction

Although since long it is well-known that extensive Boltzmann-Gibbs statistics fails to study the physical systems where (i) long-range microscopic interactions (ii) long-range memory effects are present and (iii) the system evolves in a multifractal space-time, a tendency towards the nonextensive formalisms keeps growing nowadays. One of these formalisms is the quantum groups (QG) [1] whereas the other is the so-called Tsallis Thermostatistics (TT) [2]. Before summarizing TT, it is worthwhile to note that although these two formalisms seem to be very distinct, some recent developments [3] indicate that there is a connection between them. Hence, further developments on this line by studying systems within both formalisms would be expected.

In order to summarize the TT, let us recall the axioms on which the formalism is based.

- The entropy of the system is defined to be

$$S_q = -k\frac{1-\sum_{i=1}^{W} p_i^q}{1-q} \quad (q \in \Re)$$

where k is a positive constant, p_i is the probability of the system in a microstate, W is the total number of configurations and q is a new parameter which is often called the *entropic index* and it is associated with the nonextensivity of the system (it must be noted that, in general, this q and the other q appearing in QG are not identical, therefore in order to distinguish these two q's we'll use a subscript G for the q index of QG). Note that this entropy is nonextensive and recovers the well-known Shannon entropy $S_1 = -k_B \sum_i p_i \log p_i$ if and only if $q = 1$.

- q-expectation value of an observable O is given by

$$\langle O \rangle_q = \sum_{i=1}^{W} p_i^q O_i$$

which again recovers the conventional expectation value for $q = 1$.

Besides these simple axioms, the formalism also has some important properties such as existence of partition function, preservation of Legendre-structure of thermodynamics, stability, q-invariant Ehrenfest theorem, *etc.*, which makes it possible to study the physical systems within this formalism. The details of the properties can be found elsewhere [4]. It is seen that from the year 1988 up to present days the TT not only has been applied to various concepts of thermostatistics [5–22], but also achieved to solve some physical systems where Boltzmann-Gibbs statistics is known to fail. Amongst them, stellar polytropes [23], Levy-like anomalous diffusions [24], two-dimensional Euler turbulence [25], solar neutrino problem [26] and velocity distributions of galaxy clusters [27] could be enumerated.

In this manuscript, our motivation is to introduce the coherent states (CS) within TT, since, as far as we know, there has been no attempt on this line yet. To do this, we generalize the Landau diamagnetism by using the base of CS, which has exhibited the property that it makes the calculations very easy compared to the other bases. In addition to this, the effect of nonextensivity has been illustrated with the magnetization curve. In the last part of

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this paper, the same subject (*i.e.* Landau diamagnetism) has been attempted to be considered within QG, with the aim of establishing relations between TT and QG by studying Landau diamagnetism in the frame of both formalisms.

2 Coherent states of an electron in a uniform magnetic field

The Hamiltonian of a free electron in a magnetic field H, neglecting spin, is given by [28]

$$\mathcal{H} = \frac{\pi^2}{2m} \tag{1}$$

where m is the electron mass and

$$\boldsymbol{\pi} = \mathbf{p} + \frac{e}{c}\mathbf{A} \tag{2}$$

$$\mathbf{H} = \boldsymbol{\nabla} \wedge \mathbf{A}. \tag{3}$$

We choose the vector potential to be

$$\mathbf{A} = \left(-\frac{1}{2}Hy, \frac{1}{2}Hx, 0\right) \tag{4}$$

so that $\mathbf{H} = H\mathbf{k}$. Thus, the Hamiltonian of a free electron is written down as

$$\mathcal{H} = \frac{1}{2m}\left[\left(\mathrm{p}_x - \frac{1}{2}m\omega y\right)^2 + \left(\mathrm{p}_y + \frac{1}{2}m\omega x\right)^2\right] \tag{5}$$

where $\omega = eH/mc$ is the electron cyclotron frequency.

Introducing the operators

$$\pi_\pm = \mathrm{p}_x \pm i\mathrm{p}_y \pm \left(i\hbar/2\ell^2\right)(x \pm iy) \tag{6}$$

where $\ell = (\hbar/m\omega)^{\frac{1}{2}}$ is the classical radius of the ground-state Landau orbit and noting that they obey the commutation relation

$$[\pi_-, \pi_+] = 2m\hbar\omega \tag{7}$$

it is easy to obtain

$$\mathcal{H} = \left(\frac{\pi_+\pi_-}{2m}\right) + \frac{1}{2}\hbar\omega. \tag{8}$$

Let the coherent state $|\alpha,\xi\rangle$ of this system to be defined by the simultaneous eigenstate of the two commuting non-Hermitian operators which annihilate the ground state [28]:

$$\pi_- |\alpha,\xi\rangle = \frac{\hbar}{i}\frac{\alpha}{\ell^2}|\alpha,\xi\rangle, \qquad \mathrm{X}_+|\alpha,\xi\rangle = \xi|\alpha,\xi\rangle \tag{9}$$

where α and ξ are complex numbers which have dimensions of lenght and $\mathrm{X}_+ = \left(x - \frac{\pi_y}{m\omega}\right) + i\left(y + \frac{\pi_x}{m\omega}\right)$ with $[\mathcal{H}, \mathrm{X}_\pm] = 0$, $\mathrm{X}_- = \mathrm{X}_+^\dagger$. The coherent state $|\alpha,\xi\rangle$ has the following normalization condition:

$$\langle\alpha,\xi|\alpha,\xi\rangle = 1. \tag{10}$$

Moreover, CS form a complete basis and the closure relation can be expressed as

$$\frac{1}{4\pi^2\ell^4}\int |\alpha,\xi\rangle\langle\alpha,\xi|\,\mathrm{d}^2\alpha\mathrm{d}^2\xi = 1 \tag{11}$$

where $\mathrm{d}^2\alpha = \mathrm{d}\alpha_1\mathrm{d}\alpha_2$, $\mathrm{d}^2\xi = \mathrm{d}\xi_1\mathrm{d}\xi_2$. Similar to the harmonic oscillator case, these CS have also minimum uncertainty, namely,

$$\Delta x\Delta p_x = \Delta y\Delta p_y = \frac{\hbar}{2}\cdot \tag{12}$$

3 Generalization of the Landau diamagnetism and diamagnetic susceptibility

CS permit us to use the classical concepts for describing electron orbits, but yet contain all quantum effects. This approach is used to calculate the generalized partition function and the generalized Landau diamagnetism is evaluated out of it. It is also worth noting that, compared to the other bases, the use of the base of CS has been found to be quite tractable for the calculations.

In the frame of TT, the generalized partition function is given by

$$Z_q = \mathrm{Tr}\left\{[1-(1-q)\,\beta\mathcal{H}]^{\frac{1}{1-q}}\right\} \tag{13}$$

where $\beta = 1/kT$. In the base of CS, in order to evaluate Z_q for a cylindrical body of length L, radius R, oriented along the magnetic field, we can separate the partition function into two parts [28]:

$$Z_q = Z_\parallel Z_\perp \tag{14}$$

where $Z_\parallel$ is the partition function which is parallel to the cylinder and $Z_\perp$ is the transverse part. Here, it is worth noting that the factorization used above is an approximate scheme which has been used first in [8] and shown to be useful in [17,19]. $Z_\parallel$ and $Z_\perp$ are defined as

$$Z_\parallel = \left(\frac{L}{h}\right)\left(\frac{2\pi m}{\beta}\right)^{\frac{1}{2}}, \tag{15}$$

$$Z_\perp = \int \frac{\mathrm{d}^2\xi\mathrm{d}^2\alpha}{4\pi^2\ell^4}\langle\alpha,\xi|\left[1-\frac{\beta}{2m}(1-q)\,\pi_+\pi_-\right]^{\frac{1}{1-q}}|\alpha,\xi\rangle. \tag{16}$$

The transverse part, equation (16), can be simplified through the properties of CS. $[...]^{\frac{1}{1-q}}$ term at the right hand side of equation (16) can be expanded as

$$\left[1-\frac{\beta}{2m}(1-q)\,\pi_+\pi_-\right]^{\frac{1}{1-q}}=$$
$$1-\frac{\beta}{2m}\pi_+\pi_-+\frac{1}{2!}\left(\frac{\beta}{2m}\pi_+\pi_-\right)^2 q$$
$$-\frac{1}{3!}\left(\frac{\beta}{2m}\pi_+\pi_-\right)^3 q\,(2q-1)+\cdots$$
$$=\sum_{n=0}^{\infty}\frac{\left\{[1-\hbar\omega\beta(1-q)]^{\frac{1}{1-q}}-1\right\}^n}{(2m\hbar\omega)^n\,n!}\pi_+^n\pi_-^n\,. \quad (17)$$

By substituting equation (17) in equation (16), using equation (9) one can easily obtain

$$Z_\perp=$$
$$\int\frac{\mathrm{d}^2\xi\mathrm{d}^2\alpha}{4\pi^2\ell^4}\sum_{n=0}^{\infty}\frac{\left\{[1-\hbar\omega\beta(1-q)]^{\frac{1}{1-q}}-1\right\}^n}{(2m\hbar\omega)^n\,n!}\left(\frac{\hbar^2|\alpha|^2}{\ell^4}\right)^n$$
$$=\frac{1}{4\pi^2\ell^4}\int\exp\left\{-\frac{|\alpha|^2}{2\ell^2}\left[1-(\hbar\omega\beta(1-q))^{\frac{1}{1-q}}\right]\right\}\mathrm{d}^2\xi\mathrm{d}^2\alpha. \quad (18)$$

In order to calculate the integral of equation (18) over α and ξ, let us exclude all CS with $|\alpha+\xi|>R$, *i.e.*, we sum over all orbits lying within the cylinder. If $R\gg\ell$, the exponential term rapidly converges with $|\alpha|^2$ [28]. Therefore we can safely expand the integration over α to all complex plane. More precisely, one can write down

$$Z_\perp=\frac{1}{4\pi^2\ell^4}\int_0^R 2\pi|\xi|\,\mathrm{d}|\xi|\times$$
$$\int_0^\infty 2\pi|\alpha|\,\mathrm{d}|\alpha|\exp\left\{-\frac{|\alpha|^2}{2\ell^2}\left[1-(1-\hbar\omega\beta(1-q))^{\frac{1}{1-q}}\right]\right\}$$
$$=\frac{(R/\ell)^2}{1-[1-\hbar\omega\beta(1-q)]^{\frac{1}{1-q}}}\cdot \quad (19)$$

Substituting equation (19) in equation (14), the generalized partition function of a free electron gas in a uniform magnetic field can be obtained as

$$Z_q=V\frac{(2\pi m/\beta)^{1/2}}{h}\frac{m\omega}{4\pi\hbar}\frac{1}{1-[1-\hbar\omega\beta(1-q)]^{\frac{1}{1-q}}}\cdot \quad (20)$$

Adding the zero-point energy, it is straightforward to write down the generalized free energy

$$F_q=\frac{n\hbar\omega}{2}-\frac{n}{\beta}\frac{Z_q^{1-q}-1}{1-q} \quad (21)$$

and the generalized magnetization

$$M_q=-\frac{\partial F_q}{\partial H}$$
$$=\frac{ne\hbar}{mc}\left[-\frac{1}{2}+Z_q^{1-q}\left(\frac{kT}{\hbar\omega}-\frac{1}{X}(1-X)^q\right)\right] \quad (22)$$

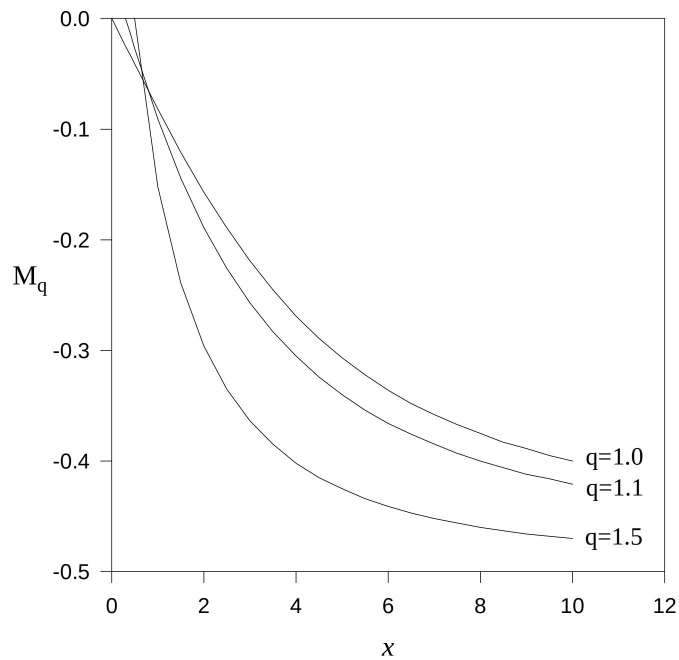

Fig. 1. The magnetization *versus* dimensionless variable x for various values of q.

where $X=1-[1-(1-q)\hbar\beta\omega]^{\frac{1}{1-q}}$. It is easy to verify that in the $q\to 1$ limit, equation (22) leads to the standard magnetization [28]

$$M_1=\frac{ne\hbar}{mc}\left(\frac{k_{\mathrm{B}}T}{\hbar\omega}-\frac{1}{2}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\cdot \quad (23)$$

The generalized susceptibility per electron, χ_q , in the high temperature limit can be obtained as

$$\chi_q=-\frac{1}{n}\frac{\partial M_q}{\partial H}=\left(3q^3+2q^2-4q\right)\chi_1 \quad (24)$$

where

$$\chi_1=-\frac{1}{3}\left(\frac{e\hbar}{2mc}\right)^2\frac{1}{k_{\mathrm{B}}T} \quad (25)$$

which is nothing but the standard diamagnetic susceptibility.

As a consequence of equation (24), there exist a special $q^*=0.87$ value, below which the generalized susceptibility vanishes. This special q^* value exhibits an interesting analogy with the value $q^*=0.85$, which has been found by Cannas and Tsallis [29], for a completely different magnetic system (Ising ferromagnet).

In Figure 1, the generalized magnetization *versus* dimensionless variable $x\equiv\hbar\omega/kT$, for various values of q, has been illustrated. The figure indicates clearly that a small deviation in extensivity (namely, the system becomes slightly nonextensive) causes the magnetization to change considerably. This fact is exactly the same as what appeared in some other systems [18,19,22] which have been investigated within TT. It is seen that the magnetization goes to zero for a high but finite temperature.

This unconventional situation could be considered as that the system, in some sense, exhibits a kind of phase transition, due to the effect of nonextensivity. More precisely, the system is diamagnetic up to zero x value, then it somehow becomes paramagnetic (since the values of magnetization become positive).

4 The effect of a harmonic potential on the generalized diamagnetic susceptibility

Let us now consider the case where the system is perturbed by an external force. Thus, a harmonic potential term with ω_0 frequency is added to the Hamiltonian (8), which gives

$$\mathcal{H} = \frac{1}{2}\left[\frac{\pi_+\pi_-}{2m}\left(1+\frac{\omega}{\omega'}\right) + \frac{1}{2}m\omega'^2 \mathrm{X}_-\mathrm{X}_+\left(1-\frac{\omega}{\omega'}\right) + \hbar\omega'\right] \quad (26)$$

where $\omega' = \left(\omega^2 + 4\omega_0^2\right)^{1/2}$. Note that in the definitions of ℓ, $\mathrm{X}_\pm$ and $\pi_\pm$, ω is replaced by ω'.

By taking into account only the transverse part of the partition function, the generalized partition function is given by

$$Z_q = \int \frac{\mathrm{d}^2\xi \mathrm{d}^2\alpha}{4\pi^2\ell^4}\left\langle \alpha,\xi \left| [1-(1-q)\beta\mathcal{H}]^{\frac{1}{1-q}} \right| \alpha,\xi \right\rangle . \quad (27)$$

To calculate the integral in equation (27), $[...]^{\frac{1}{1-q}}$ term can be factorized into two terms in the sense of the factorization scheme recently defined and used in similar calculations [19,17,8]. At this point, it is also worthwhile to emphasize that since CS have a minimum uncertainty, in this base $\langle AB\rangle = \langle A\rangle\langle B\rangle$, which yields the calculations to become tractable compared to other bases. Hence, equation (27) turns out to be

$$\begin{aligned} Z_q = \int &\left\langle \alpha,\xi \left| \left[1-\frac{\beta}{4m}\left(1+\frac{\omega}{\omega'}\right)(1-q)\pi_+\pi_-\right]^{\frac{1}{1-q}} \right| \alpha,\xi \right\rangle \\ \times &\left\langle \alpha,\xi \left| \left[1-\frac{\beta m\omega'^2}{4}\left(1-\frac{\omega}{\omega'}\right)(1-q)\mathrm{X}_-\mathrm{X}_+\right]^{\frac{1}{1-q}} \right| \alpha,\xi \right\rangle \\ \times &\frac{\mathrm{d}^2\xi\mathrm{d}^2\alpha}{4\pi^2\ell^4} = \int_0^\infty\int_0^\infty \exp\left\{-\frac{|\alpha|^2}{2\ell^2}\left[1-\left(1 - \frac{\hbar\beta}{2}(\omega'+\omega)(1-q)\right)^{\frac{1}{1-q}}\right]\right\} \\ \times &\exp\left\{-\frac{|\xi|^2}{2\ell^2}\left[1-\left(1-\frac{\hbar\beta}{2}(\omega'-\omega)(1-q)\right)^{\frac{1}{1-q}}\right]\right\} \\ \times &\frac{2\pi|\xi|\mathrm{d}|\xi| 2\pi|\alpha|\mathrm{d}|\alpha|}{4\pi^2\ell^4} . \end{aligned} \quad (28)$$

By taking into consideration $\hbar\omega'/2$ term in equation (26), one can finally find

$$\begin{aligned} Z_q = &\left[1-\frac{\hbar\omega'\beta}{2}(1-q)\right]^{\frac{1}{1-q}} \\ &\times\left\{\left[1-\left(1-\frac{\hbar\beta}{2}(\omega'+\omega)(1-q)\right)^{\frac{1}{1-q}}\right]\right. \\ &\times\left.\left[1-\left(1-\frac{\hbar\beta}{2}(\omega'-\omega)(1-q)\right)^{\frac{1}{1-q}}\right]\right\}^{-1} . \end{aligned} \quad (29)$$

Note that the standard result

$$Z_1 = \left[4\sinh\frac{\hbar(\omega'-\omega)\beta}{4}\sinh\frac{\hbar(\omega'+\omega)\beta}{4}\right]^{-1} \quad (30)$$

can be obtained in the $q\to 1$ limit.

For $\omega_0 \ll \omega$, the generalized susceptibility reads

$$\chi_q = \left(3q^3+2q^2-4q\right)(2\omega_0)^{q-1}\chi_1. \quad (31)$$

From this expression, it is observed that in the $q\to 1$ limit there is no contribution to the susceptibility from the oscillator term, whereas for $q\neq 1$ case there is a contribution to the susceptibility in the order of $(2\omega_0)^{q-1}$. It is interesting to see that the nonextensive effects change the contribution of the perturbation term to the diamagnetic susceptibility.

5 q_G-deformed Landau diamagnetism

Parallel to the rapid increase of the interest towards quantum groups recently, the investigations on the deformation of the quantum harmonic oscillator systems (bosonic and fermionic) have also been progressing fairly well [1]. Although the q_G-deformed algebra has been throughly investigated in the referred articles, it is instructive to give here a brief review.

q_G-bosonic oscillator algebra is defined by the commutation relations

$$\mathrm{aa}^+ - q_\mathrm{G}\mathrm{a}^+\mathrm{a} = q_\mathrm{G}^{-N}, \quad [\mathrm{N},\mathrm{a}] = -\mathrm{a}, \quad \left[\mathrm{N},\mathrm{a}^+\right] = \mathrm{a}^+ \quad (32)$$

where q_G is the deformation parameter, a, a^+ and N are the annihilation, creation and number operators, respectively. Eigenstates $|n\rangle$ of the number operator N are represented in the q_G-Fock space as

$$|n\rangle = \frac{(\mathrm{a}^+)^n}{\sqrt{[n]!}}|0\rangle, \quad (n=0,1,2,\cdots) \quad (33)$$

where $[n]! \equiv [n][n-1]\cdots[1]$, $[0]! = 1$ and $[n]$ is given by

$$[n] = \frac{q_\mathrm{G}^n - q_\mathrm{G}^{-n}}{q_\mathrm{G} - q_\mathrm{G}^{-1}} . \quad (34)$$

It could be noticed that this equality is invariant under $q_{\mathrm{G}} \leftrightarrow 1/q_{\mathrm{G}}$.

In the q_{G}-Fock space, it is known that the following relations hold:

$$\mathrm{a}^{+}\mathrm{a} = [\mathrm{N}], \quad \mathrm{a}\mathrm{a}^{+} = [\mathrm{N}+1]. \tag{35}$$

Lastly, it is emphasized that the actions of a, a^{+} and N on $|n\rangle$ are given by

$$\begin{aligned} \mathrm{a}\,|n\rangle &= [n]^{1/2}\,|n-1\rangle \\ \mathrm{a}^{+}\,|n\rangle &= [n+1]^{1/2}\,|n+1\rangle \\ \mathrm{N}\,|n\rangle &= n\,|n\rangle\,. \end{aligned} \tag{36}$$

The magnetization of an electron gas in weak magnetic fields is made up (i) a paramagnetic part due to the spin magnetic moments of the electrons and (ii) a diamagnetic part due to the quantization of the orbital motion of the electrons in the magnetic field (Landau diamagnetism). The detailed investigations of the subject can be found in the standard textbooks of Statistical Physics [30].

In order to express the q_{G}-deformed Landau diamagnetism, let us start by considering the q_{G}-oscillator Hamiltonian

$$\mathcal{H}_{q_{\mathrm{G}}} = \frac{P_{q_{\mathrm{G}}}^{2}}{2m} + \frac{m\omega^{2}}{2}X_{q_{\mathrm{G}}}^{2}. \tag{37}$$

By using equations (35, 36), this expression can be written down as

$$\mathcal{H}_{q_{\mathrm{G}}} = \frac{\omega}{2}\left(\mathrm{a}^{+}\mathrm{a} + \mathrm{a}\mathrm{a}^{+}\right) = \frac{\omega}{2}\left([\mathrm{N}] + [\mathrm{N}+1]\right) \tag{38}$$

where $\hbar = 1$. This Hamiltonian is diagonal in the base $|n\rangle$ and its eigenvalues are

$$E_{n}\left(q_{\mathrm{G}}\right) = \frac{\omega}{2}\left([n] + [n+1] - 1\right) \tag{39}$$

where the zero point energy is translated to the origin of the energies. In the $q_{\mathrm{G}} \to 1$ limit, $E_{n}\left(q_{\mathrm{G}} \to 1\right) = \omega n$. Then q_{G}-deformed partition function of the oscillator is defined to be [31]

$$\begin{aligned} \left(Z_{\mathrm{osc}}\right)_{q_{\mathrm{G}}} &= \sum_{n=0}^{\infty} \exp\left[-\beta E_{n}\left(q_{\mathrm{G}}\right)\right], \\ \beta &= \frac{1}{T} \quad \left(k_{\mathrm{B}} = 1\right). \end{aligned} \tag{40}$$

Now, let us consider a unit volume of the electron gas in a homogenous magnetic field H which is oriented in the direction of the z-axis. The q_{G}-deformed partition function for this system is

$$\left(Z_{\mathrm{el}}\right)_{q_{\mathrm{G}}} = \frac{m\omega}{2\pi}\left(\frac{2\pi\beta}{m}\right)^{1/2}\left(Z_{\mathrm{osc}}\right)_{q_{\mathrm{G}}} \tag{41}$$

where m is the electron mass and $\omega = eH/mc$. Along the lines of reference [28], the q_{G}-deformed magnetization can be found as

$$M_{q_{\mathrm{G}}} = -\frac{\partial F_{q_{\mathrm{G}}}}{\partial H} = -\frac{e}{mc} \left\{ \frac{1}{2} - \frac{1}{\beta\omega} - \frac{\frac{1}{2}\sum_{n=0}^{\infty}\left([n] + [n+1] - 1\right)\mathrm{e}^{-\beta E_{n}(q_{\mathrm{G}})}}{\sum_{n=0}^{\infty}\mathrm{e}^{-\beta E_{n}(q_{\mathrm{G}})}} \right\}. \tag{42}$$

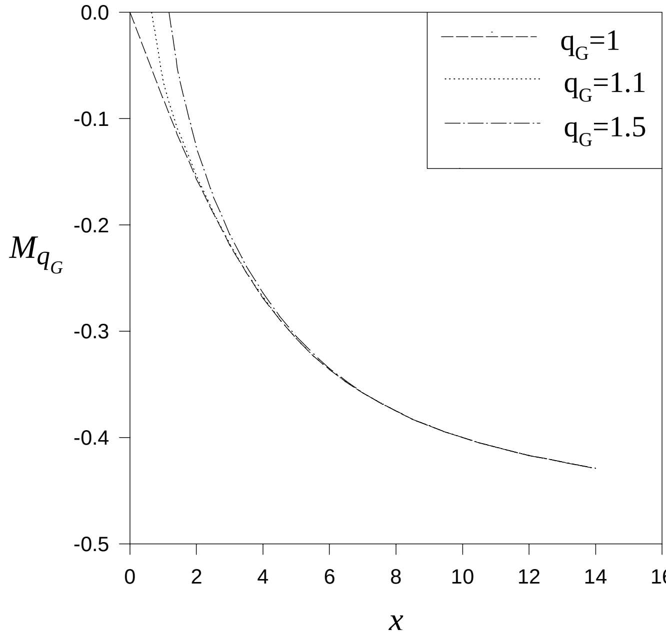

Fig. 2. The magnetization *versus* dimensionless variable x for various values of q_{G}.

One could easily check that all the q_{G}-deformed expressions derived above transform to well-known standard results in the limit of $q_{\mathrm{G}} \to 1$. The q_{G}-deformed Landau diamagnetism *versus* the dimensionless quantity $x \equiv \omega/T$, for different q_{G} values has been given in Figure 2. It is observed that for low temperatures (*i.e.*, for large x) the curves of various q_{G} values are the same, whereas for high temperatures (*i.e.*, for small x) plots start to deviate from each other and for a certain very high temperature, unlike the classical case, the magnetization tends to zero and for temperatures higher than this value, similar to what has been observed in Figure 1, the system exhibits a phase transition-like feature, passing from diamagnetic case to paramagnetic case. This tendency of the magnetization curve is the same as the one obtained in Figure 1, indicating the relation between two formalisms once again. On the other hand, most interesting of all, magnetization curves in Figure 2 are completely in good agreement with the remarks of Tsallis in [3]. Here $q_{\mathrm{G}} < 1$ case has not been considered since $q_{\mathrm{G}} \leftrightarrow 1/q_{\mathrm{G}}$.

6 Conclusions

In the first part of this study, Landau diamagnetism has been generalized within TT by introducing CS for the first time (as far as we know) in the frame of this formalism. From the generalized magnetization, the effect of nonextensivity has clearly been illustrated, similar to what appeared in some recent works on different systems [18, 19,22]. Although q is equal to unity for the conventional Landau diamagnetism, it might be useful to consider the $q \neq 1$ case whenever the system is thought to evolve in a fractal-like space-time. In addition to this, the effect of a perturbation term on diamagnetic susceptibility has been obtained by adding a harmonic potential to the Hamiltonian. It is worthwhile to emphasize that for $\omega_0 \ll \omega$, no contribution comes from the harmonic perturbation term to the diamagnetic susceptibility in the $q \to 1$ limit, whereas in the $q \neq 1$ case, there is a contribution in the order of $(2\omega_0)^{q-1}$, which is a consequence of the nonextensive effects.

Since the interest on nonextensive physical systems is progressing along two main lines namely TT and QG, in the last part of this paper, with the help of the partition function of q_G-deformed oscillator, the q_G-deformed Landau diamagnetism has been investigated. It is seen from the magnetization curves that the effect of nonextensivity is similar to that obtained from TT, moreover, it is evident that the behaviour of the curves completely agrees with the remarks of Tsallis [3], a rather pleasant result.

Summing up, in the recent years, some physical systems such as harmonic oscillators [5,32], symmetric top [33,34], blackbody radiation [18,19,35] and dark magnetism problem [36,37] have been handled in the frame of TT and QG; and in this paper, we contribute to this field by investigating the Landau diamagnetism within these two formalisms.

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